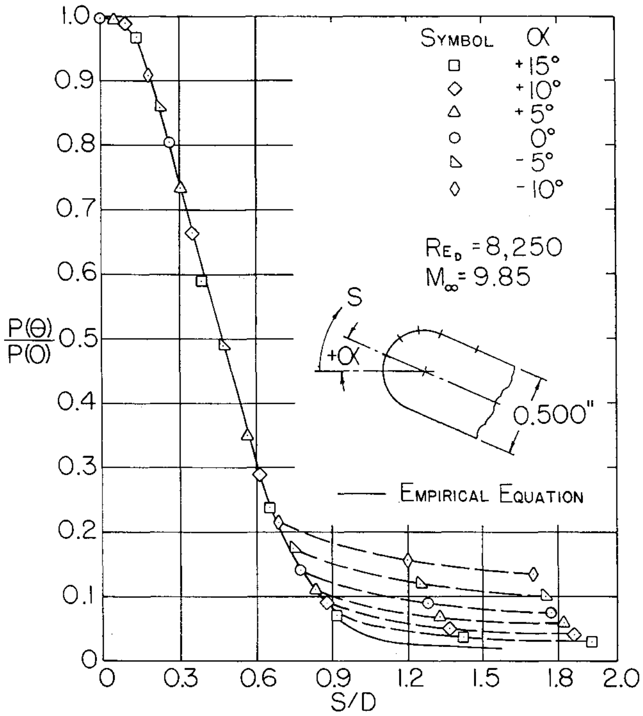

FIG. 3. Pressure distribution about the nose region of a blunt flat plate.

little dependence of the nose pressure distribution upon the afterbody shape for the conditions tested.

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Euler Load of a Stepped Column—An Exact Formula

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SYMBOLS

E_iI_i = rigidity of ith section
l_i = length of ith section
$k_i^2 = P/E_iI_i$
$S_i = k_i \tan k_il_i$
$T_i = \tan k_il_i/k_i$

THE EULER buckling load of the uniform column with the end conditions shown in Fig. 1 is the (lowest) root of the transcendental equation[1]

$$T = l \qquad (1)$$

The method[1] can be extended to the case of the stepped column shown in Fig. 2. For $\eta > 2$ the method is laborious. However it is possible to obtain the corresponding transcendental equation for general n; viz.,

$$\sigma_1 - \sigma_3 + \sigma_5 - \ldots = (1 - \sigma_2 + \sigma_4 - \ldots)\sum_1^n l_i \qquad (2)$$

where

$$\sigma_{2r+1} = \Sigma T_iS_jT_k\ldots S_lT_m \qquad 2r + 1 \text{ factors} \quad i<j<k<\ldots<l<m$$

and

$$\sigma_{2r} = \Sigma S_iT_j\ldots S_kT_l \qquad 2r \text{ factors} \quad i<j<\ldots<k<l$$

Eq. (2) may be solved numerically with the aid of tables of the tangent function.

By this method we obtain the Euler buckling load without, in effect, solving for the buckled shape of the column. Again, we obtain the "exact" Euler load and, lastly, the form adduced in Eq. (2) is general. All these are advantages over any energy method.

When n is any specified integer, direct (and laborious) extension of the method[1] can be shown to lead to Eq. (2). However eq. (2) is true for general n.

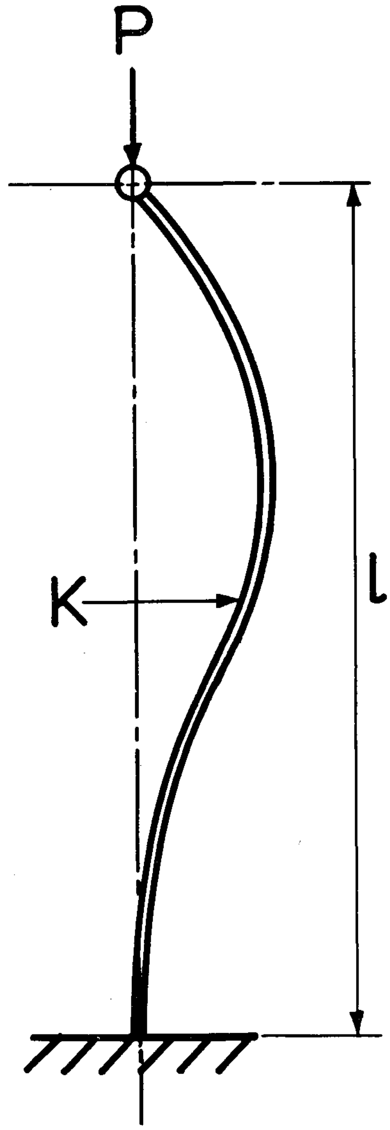

FIG. 1.

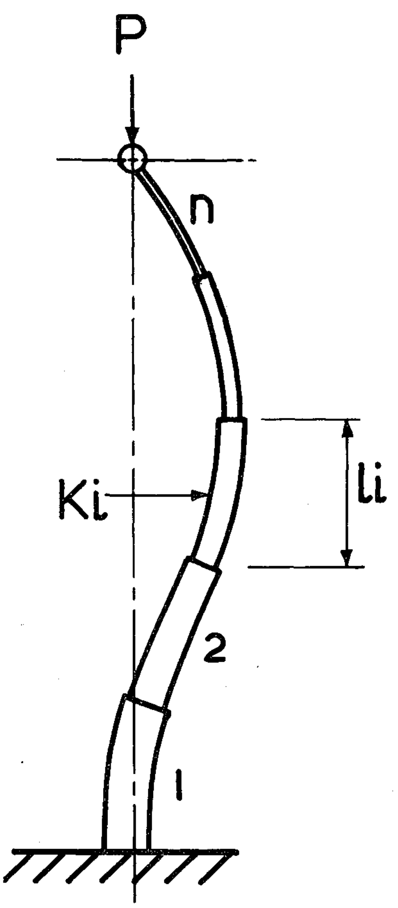

Fig. 2.

Reference

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A Formula for Certain Types of Stiffness Matrices of Structural Elements

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September 24, 1962

The linear theory is assumed throughout. The procedure is outlined for the three-dimensional case. Let

$$\left.\begin{aligned} k &= \text{col}\,(k_1, k_2, \ldots, k_m) \\ \sigma &= \text{col}\,(\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx}) \\ \epsilon &= \text{col}\,(\epsilon_x, \epsilon_y, \epsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{zx}) \end{aligned}\right\} \quad (1)$$

where k is a vector of constants, σ the stress vector and ϵ the strain vector at some point P in the element. The stress and strain vectors are related by an equation of the form

$$\epsilon = N\sigma \quad (2)$$

where N is a nonsingular symmetric matrixi nvolving Poisson's ratio and Young's modulus. We let

$$\sigma = Uk \quad (3)$$

where U is a matrix whose elements are of the form $x^{\nu_1}y^{\nu_2}z^{\nu_3}$ with the ν_i integers or zero, so that each stress is assumed approximated by a polynomial whose coefficients are components of the vector k. The polynomials are chosen to satisfy the differential equations of equilibrium and compatibility.

Following somewhat the general plan of Refs. 3 and 4 the stresses are integrated over the surface of the element giving a relation between the applied loads p, the resultants of these stresses, and the vector k.

$$p = Vk \quad (4)$$

Since the stresses satisfy the conditions of equilibrium the element is also in equilibrium under the loads p. The case discussed here is special in that the applied loads are assumed to determine the vector k and hence the stresses and conversely. Hence rank $(V) = m$.

For convenience of notation the first m rows of V are assumed linearly independent, hence a partitioning of Eq. (4) gives

$$\begin{pmatrix} p_1 \\ \hline p_2 \end{pmatrix} = \begin{pmatrix} V_1 \\ \hline V_2 \end{pmatrix} k \quad (5)$$

where V_1 is nonsingular. From this one obtains

$$p_1 = V_1 k \qquad p_2 = V_2 k \quad (6)$$

giving

$$k = V_1^{-1} p_1 \quad (7)$$

Hence p_1 alone determines k and the components of p_1 may be taken as independent variables. Fixing the displacements corresponding to p_2 the element becomes supported.

Now consider the flexibility matrix of the supported element. In Ref. 1 the following formula is given:

$$f_{i,j} = \int_V \epsilon^{iT} \sigma_j dV \quad (8)$$

where ϵ^i = the column strain vector at point P in the element of volume V due to a unit load at coordinate i. ϵ^{iT} = the transpose of ϵ^i, σ_j = the column stress vector at P due to a unit load at j. Combining Eq. (3) with (7) and letting e_j = a column vector with 1.0 in the jth place and zeros elsewhere gives

$$\sigma^j = UV_1^{-1}e_j \quad (9)$$

from which by use of Eq. 2 one obtains

$$\epsilon^{iT} = (N\sigma^i)^T = \sigma^{iT}N = e_i^T V_1^{-1T} U^T N \quad (10)$$

Inserting in Eq. (8) and noting that only the components of the U matrix are affected by the integration gives

$$f_{i,j} = e_i^T V_1^{-1T} \left(\int_V U^T N U dV\right) V_1^{-1} e_j = e_i^T V_1^{-1T} G V_1^{-1} e_j \quad (11)$$

where

$$G = \int_V U^T N U dV \quad (12)$$

But (11) amounts to the matrix equation

$$F = (f_{i,j}) = V_1^{-1T} G V_1^{-1} \quad (13)$$

We now show that the matrix G is nonsingular. The energy stored under any loading is, by Ref. 2,

$$U = {}^1/_2 \int_V \epsilon^T \sigma dV \quad (14)$$

which by use of Eqs. 2 and 3 becomes

$$U = {}^1/_2 \int_V \sigma^T N \sigma dV = {}^1/_2 \int_V k^T U^T N U k dV = {}^1/_2 k^T G k \quad (15)$$

Now stored energy can be zero only if the loading is zero, which implies that the vector k is also zero. Otherwise it is positive. This amounts to saying that G is positive-definite and hence nonsingular.

By the inversion of Eq. (13) one obtains the $m \times m$ stiffness matrix of the supported element

$$\bar{S} = V_1 G^{-\gamma} V_1^T \quad (16)$$

To obtain the unsupported $n \times n$ stiffness matrix S we define the load transformation matrix H to satisfy

$$\begin{pmatrix} p_1 \\ \hline p_2 \end{pmatrix} = H p_1 \quad (17)$$

so that